

Indian Statistical Institute, Bangalore
B. Math (Hons.) Second Year
Second Semester - Ordinary Differential Equations
Supplementary Examination

Total marks: 50 **Maximum marks:** 45

Date: June 3, 2026

(Any score above 45 will be treated as 45)

Duration: 3 hours

ANSWER ALL QUESTIONS

1. Solve the following equations:

(a) $(y \log y - 2xy)dx + (x + y)dy = 0.$ [5]

(b) $(x \sin \frac{y}{x}) \cdot y' = y \sin \frac{y}{x} + x.$ [5]

2. Consider the second order linear differential equation: [10]

$$3y''(t) - 15y'(t) + 18y(t) = 0$$

(a) Find the general solution.

(b) Find the particular solution satisfying $y(0) = 0, y'(0) = 1.$

(c) When is $\lim_{t \rightarrow \infty} y(t) = 0$?

3. Plot the phase potrait and classify the fixed points of the following systems:

(a) $\dot{x} = -3x + 4y, \dot{y} = -2x + 3y$ [5]

(b) $\dot{x} = x(x - y), \dot{y} = y(2x - y)$ [5]

4. Show that the system $\dot{x} = -x - y + x(x^2 + 2y^2), \dot{y} = x - y + y(x^2 + 2y^2)$ has at least one periodic solution. [5]

5. Solutions of *Airy's equation* $y'' + xy = 0$ are called *Airy functions*.

(a) Verify that every nontrivial Airy function has infinitely many positive zeros and at most one negative zero. [5]

(b) Find the Airy functions in the form of power series, and verify directly that these series converge for all x . [5]

(c) Use the results of (b) to write down the general solution of $y'' - xy = 0$ without calculation. [5]

Good luck!!